# Teaching Mathematics with Technology (TMT)

# Video - Use of Geogebra for Teaching Learning – Basic Proportionality theorem

**Opening webcam/ Slide 1**

Hello, and welcome back to our video series on exploring teaching-learning with Geogebra. In our previous video, we had looked at the concept of similarity, and we had worked through that concept through a series of sketches starting with the idea of congruent objects, congruent triangles, establishing what are corresponding sides and corresponding angles looked at proportionality and then looked at the idea of similar triangles.

We also left the students with the idea of what are the parameters in similar triangles and how they behave when the two triangles are similar, in terms of the angles as well as in terms of the sides. And with this background, we are ready to now look at the basic *Proportionality Theorem.*
We will do that again through a series of Geogebra sketches itself.

**Slide 2**

We will look at the idea of *proportionality, and* we will help the students reinforce their understanding of *corresponding sides and proportionality*. We will build a generic triangle and will show the students, how the *ratios of corresponding sides* behave. We will also need to *explore the area of a triangle* further and understand what *the height* means, and therefore what we can say about *triangles between parallel lines.* Once we have done these two prior sketches and prior concept revisiting, we are ready to look at the *logical proof of the basic Proportionality Theorem* itself.

**Geogebra Construction**

We now have a sketch in front of us, which is *two rays*, and we actually have a *series of points plotted* here. I will just click this *check box* here to show the *arcs,* to sort of give you an idea of how this lesson is going to progress. And we now also see that we have a *triangle here which is AEB* and then these *arcs* have been marked. So basically, we can now draw a line through *point P parallel to line BE*, or we can draw a line at *point Q parallel to BE* and so on. And in each one of these cases, we can explore if the triangles are *similar*, and we will look at how the *ratios of the corresponding sides* are behaving.

In this sketch, we will not be looking at proving the similarity of the constructed triangles which we know how to do, and we have done it in the prior video. So we will assume that we know how to demonstrate the similarity, and we'll just go forward with the next set of steps in terms of showing the *proportionality* and also trying to tease out the understanding for the basic *Proportionality Theorem*.

So I will go back to the beginning of the sketch. So now we will draw the line BC and so we are ready to look at a *triangle AEB.* So we'll show the arcs, we need not show the arcs. These have been created using a tool called- *Create an Arc.* And which tool has been defined by us in this construction we will share with you- a *gif,* for how to create a tool of your own in Geogebra. But here we will just use the tool that has been created, to create these arcs; and the arc lengths, of course, are variable.

So if you are able to see them here you can see that the points are changing. So now we have our line, and we are now ready to start our discussion. So I can either *draw a line at P parallel to BE, or at Q, or at R* and so on. So I will do a line parallel at point D parallel to BE. So now we are ready to look at two triangles here, *the first triangle* being *ADH,* and the *second triangle* being *ABE.* It is important for students to get an idea of this visualization and then you can show the *ratios- AH to HE* and *AD to DB* is the same. And you can also invite students to explore that these two line segments are parallel- *DH is parallel to BE* because that's how we have constructed it. And when these are parallel, these triangles are similar, because we now see that the ratios are the same. You can also go on to prove that these two triangles are indeed similar, rather verify that these triangles are indeed similar, we don't prove them yet. By showing the *angles at the vertex H* and *vertex E* and showing that this is a common angle and so on, you can sort of *verify the similarity of the two triangles* that we have now constructed.

We are now only sticking to showing the ratio. So now I will uncheck all of these things. And I will now show a segment which is not parallel and they are also able to see that this *ratio AD to DB and AJ to JE* is not the same. So, that means that these two triangles are not similar, the line segments here are not parallel and then the ratios are not equal, of corresponding sides the ratios are not equal. This has just been done to show the contrast between this construction and this construction.

So now we have already seen the idea of proportionality in a very generic sense. We are now ready to look at the logical proof of the basic *Proportionality Theorem* itself. However, before we do that we will now look at the *area of a triangle* and explore how the *altitude* has to be understood *in terms of a triangle*.

So we now have here a generic triangle- *triangle ABC.* You can position it in any way that you like, so I can perhaps even try to move the triangle a little bit like this. And we will now show the line for *side AB, as a base,* and then we'll plot a point on this line. We've called it- *K,* as you can see here and show the angle. So now it's already an altitude because it is making an angle of *900*. However, we can plot this point anywhere and move it around to actually show that the altitude is when it's actually *900*. We have already calculated the area here, using the *area function* in Geogebra and that's coming to be *16.9.* And we are also seeing here *the formula* which is, *½ x base*, which in this case *the base is AB* and the measurement is *4.7 units,* and the *altitude which is CG*, I hope you can see the *point G* here, *CG is 7.3,* and that multiplied also gives you the value of *16.9.*

We will now change this triangle around a little bit. And with the same *triangle ABC*, now that we see that this *point G* is not making an angle *900.* So it is no longer the altitude. So as you can see this multiplied value and the area of the triangle are not matching. So now we will try to see *how to make this line CG an altitude.* So, I do that by moving this point out and we see that at some point when it makes *900,* it becomes the altitude again. So I'll just move it around a little bit, you can see that it is *900* now, and it is the altitude. And if you see the value here, it is again *half into CG* which is the altitude and the base which is 4.7, it is still 10. Of course, this is a calculation which is rounded off, you can verify this.

Now the important understanding here for the students is to see that the altitude need not only remain within the triangle, but it can also actually remain outside the triangle as well. You can also show this sketch with the other side as a base, say *side BC as a base,* and take another point and do the same thing and have the students verify. This sort of strengthens the understanding in the student's mind of what is the area of a triangle, what do we mean when we say the altitude of a triangle and so on.

So once we have done this, we are now ready to move to the next step of understanding what happens when two triangles are sharing the same base or let's say have the same height, which is what happens when the two triangles lie between two parallel lines, which is one of the important constructions that we will make in the proof of the basic *Proportionality Theorem.*

So now we have two *parallel lines,* that have been drawn. I will not go over the steps of drawing a parallel line, which we already know, which you've already seen. Now we will just move the side lengths here to change the triangle. It also shows you here *move D and F* to change the triangles itself and *move point C* to move the *parallel lines*, you can see that here. So I can change it. And now you can see that the areas are the same- *area of triangle ABD and area of triangle ADF are the same, the altitudes are the same- DM and FO* are the same and they are lying between two parallel lines, and therefore the distance between the two parallel lines is constant and that also constitutes the altitude of these two triangles.

We have already seen how the altitude can be outside and now we have therefore verified here visually that *when the two triangles are between two parallel lines and they have the same base the area is the same.* It follows very logically from what we have done. We can also prove it here by just writing it down in discrete steps.

With all this prior work, now we are ready to actually explore the basic *Proportionality Theorem* itself. So we have seen here a triangle that's been constructed and we know that we will do all these prior sketches. We are ready to now look at triangle ADE. Triangle ABC, that's the basic triangle and now we are looking at triangle ADE, which is similar to triangle ABC.So now we want to show what happens to the ratio of AD to DB. In this sketch we will not be going all the way up to calculating the ratios etc.; all those steps are already known to you and we have discussed them in prior videos. We will only look at the steps that are important for the students to visualize in terms of moving in the logical proof.

So we show here the triangle BDE and show the triangle CDE and we know now that these two triangles have the same area. So we've written that step here, we've also seen here the area of triangle ADE, which can be written in terms of *½ x AD x EN,* which is one altitude. And *triangle ADE* can also be written in terms of the *½ x* AE x the other altitude. Recall, that is exactly the construction that we have seen here, where we have taken two different sides and drawn the altitude and verified the area for the same triangle. And then we have also seen that the area of these two triangles, which are lying between the *two parallel segments DE and BC with a common side DE,* these two triangles will have to have the same area.

With this now we are ready to prove the basic *Proportionality Theorem.* The rest of the proof is just algebraic manipulation and I will leave it to you to complete it in any manner that you see fit for your classroom. Those algebraic steps can also be displayed within Geogebra. And you can use a checkbox to display that as well, but you can also make the students work out the algebraic steps on their own.

**Concluding Slide / Slide 3**

So we have now seen the basic *Proportionality Theorem* and we've seen all the underlying steps that have to be worked through to work through the logical proof.

For further exploration, please download the following files which have been uploaded under *week 3* and share your insights on how do you think you would explore these concepts in your classroom. Have there been other teaching points that have emerged for you? And, do you think there is a need to do any prior work or any further explorations of these concepts as well.

Thank you for joining us and see you in our next video on the lesson plan for circles using Geogebra sketches