Teaching Mathematics with Technology (TMT)

Video - Geogebra in teaching-learning Concepts in Similarity

Opening Webcam / Slide 1

Hello, and welcome back to our video series on teaching-learning with Geogebra. In this video, we'll be looking at the concept of *similarity* and understanding the properties of *similar triangles*.

Slide 2

We saw earlier the ways in which Geogebra can support conceptual learning, in terms of *visualization*, the *dynamic construction of objects* as well as possibilities for students to observe and generalize; and allowing possibilities for conversations in the classroom by which teachers can tease out students' underlying misconceptions.

Slide 3

In this Geogebra exploration, we will be looking at '***Similarity'.*** But before we begin, I would like to just pose a question, in terms of, how do we start a discussion with '*Similarity'*. *Do we start with the idea of similarity or do we start with the idea of congruence or does it not matter?*

Slide 4

In this sequence of Geogebra sketches, we will be looking at '*similarity'* along the following lines -

We will be looking at files that allow the students *to visualize the parameters that contribute to similarity,* in terms of the *same angles* and in terms of *visualizing corresponding sides.* And we work through the idea of '*proportionality'.* And we will work towards establishing an *intuitive understanding of why the different criteria for similarity will work.*

Geogebra construction

In this Geogebra sketch, we will work with a set of congruent objects, and we will try to show to the students how *congruence* is a parameter that can be observed across different kinds of objects. We don't necessarily need to start the discussion of *congruence* with the notion only of triangles.

So for instance, we start with congruent line segments, we try to move one on top of the other and demonstrate that when two objects overlap completely, in terms of all their parameters, then they are congruent. We can demonstrate that with angles, with triangles, with quadrilaterals, and across different categories of objects as well. The idea of demonstrating with angles and segments is particularly useful when we move ahead with the *idea of theorems related to congruence*.
When we talk of *corresponding parts of congruent triangles,* then it's important for students to be able to visualize *congruence* even in the *corresponding parts*. So for them to be able to see the two angles are congruent or can be congruent, it’s a useful visualization for them to work with theorems of this kind in the future.

In the next sketch, we look at the congruence of triangles. I will increase the font size a little bit for better readability. And now we see that there are three triangles. We have now moved from different objects and establishing congruence of different objects to looking at the congruence of triangles specifically. Now we will try to move the triangle △GHI over △ABC, and we find that these overlap completely. If we see the side measures and the angle measures, it is possible for us to demonstrate that congruent triangles have the *same values of sides, corresponding sides, and corresponding angles.*

Now, this figure also gives us an interesting possibility of exploring another aspect of congruence, namely that of *orientation and rotation or reflection,* to establish congruence. For instance, if you look at *△ABC and △JKL* and if we move them to close like this, we will find that it is not possible to make the triangles overlap. However, if you keep it like this, it seems to be visually clear that *△ABC* is a reflection of *△JKL* or vice versa. We can now move this triangle to bring it closer to the △JKL, and we try to reflect this triangle along this *line of symmetry;* and it is possible now to see that this triangle overlaps the *△JKL* completely. So, if we now undo this reflection, and we can now, therefore, see that the *△ABC* when it is reflected along this *line AB* will overlay on top of *△JKL;* and it is also possible for students to then see what will be the corresponding sides. Which is not very apparent when we just try to move the triangle around. So it's possible to see that *line segment AC is congruent with line segment JK line, segment BC is congruent with side KL and line segment AB is congruent with line segment JL*. So this is another way in which the congruence can be established across different figures, and we can also show how the rotation and reflection of objects can bring about the demonstration of congruence. It's possible for the visualization to be completed through rotating or reflecting an object. Even the idea of *symmetry* and identifying the *line of symmetry* for an object and understanding how the reflection will look is also a valuable geometric insight by itself.

We will now look at another sketch that will demonstrate the idea of *proportionality*. I will again increase the font size a little bit for clearer readability. So now we will look at *the second figure*. You can first see that this is a quadrilateral of not any specially defined category, but a *generic quadrilateral*. And then we now see another figure, and if you ask the students to compare these two, they are likely to say that they are the same because we have already established in the prior figure that *when the sides and the angles are the same, then the two triangles can (or the two objects) can be overlapping with each other and therefore the two objects are congruent*. And so, if you extend that idea, it is possible for students to see that these two quadrilaterals are congruent. This was *A Prime, B Prime, C Prime, D Prime* which was a congruent figure to the original *quadrilateral ABCD.* And when we look at *the third figure* which is *A double Prime, B double Prime, C double Prime, D double prime*, it is very easy for students to visualize that this is actually a larger figure; and they are perhaps likely to say that it *looks the same*. They may not exactly use the word similar because we haven't yet introduced it as a concept, but they may just say that it kind of looks like the same thing, but it's much bigger, they may use words like this. And it is good to see if we can somehow tease out the word *proportionality* when we are having this discussion based on this sketch.

Now that we have seen the idea of congruence across different kinds of objects in our first sketch, and then we have looked at exploring the congruence of triangles specifically, and then we also looked at the concept of proportionality across a generic polygon which is a quadrilateral in this case.

Now we are ready to look at what are the parameters that determine similarity and how do we go about determining or sort of inferring the parameters of similar triangles. Now in this sketch, we have two triangles; on the face of it, they look congruent. We can actually drag and drop one on top of the other and show that it is indeed congruent. By now students should be able to recognize and determine that two figures are congruent. And we can also verify it to show the angles and the sides to be the same.

And before we work further with this sketch, it's a good idea to show to the students, how this construction works. So if we change the sliders here called - *side 1, side 2,* *and side 3*, the triangles change. There is also another set of sliders above, which say, or which as the name suggests, seem to be related to *side 1 and side 2 and side 3,* except that they have another parameter called *scaling.* So when we do *side 1 scaling*, we are trying *to change side 1 of the second triangle, vis-a-vis the first triangle*. We are trying to change the side 2 of the second triangle*, vis-a-vis the first triangle*, and the same thing for side 3 as well. So this is a set of sliders which allows us to change the dimensions of the second triangle, *vis-a-vis* the first triangle.

So now when we increase the *side 1,* you notice now that the triangle is no longer there. This is also a useful discussion to have with the students to see when a triangle does not become possible. Now we change the two parameters here and we keep changing it, and then we ask students to see when does the triangle exists. And now we have two triangles, so we can ask the students to explore whether these two triangles are congruent; they are certainly *not congruent.* Are they perhaps looking similar? Are they perhaps in some manner, you know, is there any kind of connection between the two triangles? And we can ask them to evaluate it by showing the angles and showing the sides. Clearly, it doesn't look like there is any connection between the two triangles. The angle measures are different and the side measures are different as well.

Now if we change this, now what can you say about the two triangles. I will move this down a little bit so that it's clearly visible. Now you can ask the students to explore the parameters and you will see that the angles are the same and the sides, it's just seeming to be bigger it is somewhat similar to what we saw here, that the angles are the same; the object seems
bigger.

Now we can explore this property further by moving the smaller triangle and keeping it at any one of these vertices. I will uncheck the angle so that it is a clearer display, and we keep it here. And we ask students to explore, what can they say about the sides. So it seems we have already seen that the angles are the same and the sides seem to be having some kind of a relationship. So there is *7:35,* there is *6:30* and *5:25*. So, we can explore whether it is possible for us to move the direction of the conversation where the students are actually able to say that the sides are *proportional*. Now you can even show *the ratios* so that the students can actually see that the *ratios are the same*, *which means the corresponding sides are proportional*. This is especially useful to show the ratios when the numbers are not such neatly divisible numbers like 7:35 or

6:30 or 5:25 and so on. Like for instance, if we now change it to let us say 7, again the triangles no longer look similar. You can change the sides and then you can now ask the students to see whether the triangles are looking similar. And you can move it back to a smaller size and then can ask your students to observe it. Now it is clear that the triangles are similar; you can show the ratios. So, you can move it here and you can move it here and you can move it here as well.

While we are doing the triangle and moving it around you can also leave the question open for the students, as to what can they say about the *segment BC* and the *segment EF*. And to enable the discussion, you can show the angles and you can ask them to explore what can they say about *segment BC* and *segment EF* if these two angles are equal. These will actually be parallel lines; and we can leave this question with the students. This can become a useful point of departure, for us to pick up the discussion on the basic *Proportionality Theorem,* as the next concept in this discussion on *similar triangles.*

That concludes this series of the second week of teaching-learning with Geogebra.

Slide 5

So we have seen in this lesson that *similarity is something we observe visually* and it can be fitted in terms of the *object parameters*. And we also saw *what are the parameters that determine similarity* and *why.* And we worked through the idea of *proportionality*. And then we worked through different *constructions of triangles to generalize the rule for proportionality.*

Concluding Slide / Slide 6

Please be sure to download the underlying files which are listed here. And do share your reflections on what teaching insights you get from the use of these files, keeping in mind the context of your classrooms.

We will be looking at the basic *Proportionality Theorem* in our next video. And until then please be sure to practice all the Geogebra sketches that have been shared with you on the course platform.

Thank you and see you soon.