Teaching Mathematics with Technology (TMT)

Video - Geogebra sketches for introductory concepts of lines and angles

Opening Webcam

Hello, and welcome to week 2 of this course on *Teaching Mathematics with Technology*. We hope you have familiarised yourself with the course platform as well as the basics of the GeoGebra application. During this week we will be looking at creating some sketches using GeoGebra, and we will be getting introduced to different functionalities of the application.

Before that, we will be looking in this video at the pedagogical and teaching-learning possibilities that a tool like Geogebra can provide. We will be looking at the nature of mathematics teaching and some of the challenges in effective teaching-learning of mathematics and explore what role and possibilities do digital tools and especially graphic calculators like GeoGebra can bring into the teaching-learning process.

We will be doing this through the exploration of sample GeoGebra files that we will be looking at in this video, as well as upload on the course platform, for you to explore and share your reflections on the discussion forum.

Slide 1

Thank you for joining us in this series of video lessons on teaching-learning with Geogebra. In this series, we will be looking at a series of topics in geometry and how we can use Geogebra to effectively transact these topics in a classroom.

Slide 2

Before we look at the sketches itself, I would just like to present some of the mathematical competencies that we are all very familiar with, just presented here, for the sake of completeness. I would like to draw your attention to *two strands* in particular- the *Conceptual understanding* and the *Procedural fluency*. These are no longer thought of as two different strands but rather, two strands that are intertwined, that reinforce and build upon each other. An equally important objective in school education is also to build in students the ability to logically reason out the solution to a problem or a theorem, and also to be able to communicate using their mathematical knowledge and solve problems that they may encounter in their daily life. These curricular competencies have been mentioned in different mathematics curricula across the world and this particular list has been drawn from a publication called- "*Adding It Up*" by *The National Academies Press.*

Slide 3

We have also listed here a few additional references that you may find useful to look at; in particular, I want to draw your attention to the last publication which is titled: *"Learnings from using Geogebra in high schools".* A lot of the resources that we will be presenting in this course and some of the ideas that we will be sharing have been drawn from our experience of working in Bangalore, (Karnataka), a Southern state of India - in public schools, where students have been coming to high school with a lot of gap in their learning, and also who come from very marginalized and disadvantaged backgrounds. So, some of the lessons that we had to do in mathematics classes have sort of build their conceptual bases and make sure that they are ready for high school mathematics. This particular paper captures some of the experiences of working with several different schools in the country and you might find it useful to take a look at it and there may be parallels that could be possible to be drawn for your teaching context as well.

Slide 4

So, how does Geogebra support conceptual learning?

The most obvious thing that could come to all of our minds is that of the ability to visualize a concept that we are teaching. That is certainly one of the most powerful advantages of using Geogebra in a mathematics teaching setting; what is also useful is the ability that Geogebra provides us with, in terms of constructing a certain sketch or making a parameter change and showing the construction in a dynamic manner, and you can sort of building the idea, you can even build the construction or build the proof. And that possibility of working through a construction creates opportunities for conversation in a classroom, where it's possible for us as teachers to tease out some of the misconceptions that the students might be having, based on the answers that they give us when we are presenting an idea or when the construction is unfolding. It is possible to understand what might be the underlying conceptual framework from which the student is answering that question, and that might give us some clues about how we can structure the lesson further or give us an opportunity to go back and revisit some of the concepts.

Slide 5 to 13

Before we take a look at these sketches, I would like to present to you some images that we have taken from these sketches and I would like you to pause and look at these images and think of - *what might be the answers that your students might be giving to these questions and why?*

The first question is-

(i) How many angles are there?

The possible answers are listed here as well and what do you think your students will be answering this question as.

(ii) What is the appropriate notation for labeling this angle? Again a few choices have been given.

(iii) The third sketch is asking the students *to examine the three angles and compare them and see which is a larger or smaller angle or whether they are all equal.*

Do also pay attention to the wording of the choices because that also is sometimes a factor that influences the way students are able to understand and articulate concepts.

(iv) *How many angles are there in this sketch?* Again these are the choices.

Slide 14

When we had asked these questions in the classes that we had conducted, some of the student responses have been along the following lines. Many times students could not understand the idea of a rotation of an angle; so the counterclockwise and clockwise rotation and the angle formed were seen to be the same. So, there was no distinction that they could make between the interior and the reflex angle. And the arms of the angle and the relationship with the angle measure was a source of confusion for many students. Many students implicitly believed and expressed that the greater the arm length, the larger the angle. Some angles are simply not **seen** as angles, and for instance, when the leg points are not labelled, especially in the last sketch that you saw, the angles were somehow not seen. They needed to see the leg points. We also received a lot of automatic answers like vertically opposite angles or 180^0 etc. without necessarily having the conceptual understanding underlying those answers.

Partly this is because students have been introduced to many of these concepts in a very formulaic manner and often they have been encouraged to memorize. So, therefore, they were reciting some answers from their prior memorization, but not necessarily with an underlying clear understanding of the concepts involved.

Geogebra Construction

This is the first sketch that we will be looking at, one small caveat that we before we begin with is discussion is that you will see in this and the next video, a series of tools in Geogebra that have been used which have so far not been introduced in the course itself.

I would like to state here that the tutorials for learning all these features will be uploaded during week 2 and week 3 of the course itself. However, in this lecture, we will be focusing more on the lesson aspect of the Geogebra sketch than the construction, the actual construction of the sketch itself. So when we present a sketch like this to the students, what are the possibilities that we have. We could start the discussion by asking students what they see - they are likely to say they see two segments, line segments, or they may say two lines. Some of them may even talk of the measure. Now, they may also use the word *line* and *line segment* interchangeably. This doesn't necessarily mean that they don't understand the distinction, but rather that they are not used to using mathematical terms very precisely and that may also be a reason why students are answering interchangeably.

Now it is a useful opportunity for us to use this tool to also illustrate the difference between lines and line segments as well. Now you could also ask the students what else do they see here, do they see an angle? Students may recognize that there is an angle here, sometimes they may not recognize that there is an angle here. So when you show the points it's possible for them to more easily identify an angle that is visible in the sketch. And if you ask them to label the angles, they are just as likely to answer it as IDE or EDI. One of the reasons why they answer like this is that in the elementary or even at upper elementary context especially in the context that we have been working in, is that the emphasis has not been so much on the understanding of the idea of the rotation, or the clockwise or the counterclockwise rotation conventions that have been followed. Typically, we take the angle measure as being rotated counterclockwise; this is not explicitly discussed with the students.

With the use of a tool like Geogebra, it is possible for us to actually show that ∠IDE means the angle that is measured interior and ∠EDI is the *exterior* or rather the *reflex* angle that gets measured. When we are talking of a counterclockwise rotation, the direction of the rotation of the angle, the segments matters, and depending upon how you choose to measure from EDI or IDE we are physically measuring two different angles. This is a very useful point for the students to get sight of.

Yet another extension of the discussion could be in terms of moving the segments itself. I will not show the angle now. I will show the interior angle and the exterior angle and when I **move** this segment you can ask the students to observe what is happening to the angles - the interior angle as well as the reflex angle. And when the angle becomes 180^0, it is also possible to ask the students - what is that we see here. Students may again say there is a line and at that point, it becomes useful to draw a line through these two points and show that there is a line on which these two segments lie, and when they lie on the line, they make an angle 180^0. So this means the angle in a straight line is 180^0 , and therefore it is also called a straight angle, and the entire discussion can proceed along those lines.

You can also show the reflex angle formed on the other side of the straight line and it is useful for the students to see that if you take a straight line you can see an angle of 180^0 on either side of the line. At this point, I will now delete this line and I would like to demonstrate the idea of a linear pair here as well; that also becomes a useful discussion. We don't need to show the reflex angle for this demonstration. I will draw a ray from *point D*, and it is *point A* here. We need not show the ray, we can just draw the segment or you can show the ray with the point mark, that's up to you. But we will label the point here, right, and now we can ask the students what kind of angles are formed. Before we ask we could either show them the entire 180^0 or just show them this sketch and ask them what the angles are, how many angles are there? Now we can show the measurement of the angles as well. It is $\angle IDA$ *and* $\angle ADE$. I will move it around for ease of display and you can use the *caption* tool in Geogebra to label these angles as angle 1 and angle2 as constituting a *linear pair*.

You can see that there are two angles here, $\angle IDA$ *and* $\angle ADE$. $\angle 1 = 59^0$ and $\angle 2 = 121^0$, and together these constitute a straight angle or a linear pair. This becomes another possible extension of this lesson as well.

We'll now go back to our original sketch and explain one more concept or rather misconception in children in terms of the relationship between the length of the arms of the angle and the angle measure itself.

Now you can vary the *sliders* here, which are called *arm 1* and *arm 2*, and ask students to observe what is happening to the angle. So now I will move this slider by either using my keyboard or by using the mouse. And you can ask the students to observe what happens to the value of the angle as you increase the arm length, the angle doesn't change. You can also change one arm length and keep the other one different and ask the students to observe what happens to the angle. These are all useful points to illustrate because students often tend to think that the arm length is somehow directly linked with the angle measure and the longer the length of the arms - the larger the length of the arms, they think that the angle is larger. Now we can also show to them when actually the angle measure change. It does not vary based on the length of the arms of the angle but when the segments are rotated; that is when the angle changes.

Yet another useful discussion to have with this sketch is by simply changing the orientation of the angle. For instance, presenting an angle like this. Often students are used to seeing angles presented in a particular orientation, and they don't recognize an angle in a context that is different from that orientation. It is useful for them to see angles like this as well. And that it doesn't really matter how it is presented vis-a-vis the *blackboard orientation* and so on. This is another useful thing to show to the students. Now at this point, we are ready to extend this discussion to the idea of the intersection of two lines and three lines and the formation of closed space through the intersection of three lines, etc. So for that, I will use another sketch. We have a *line1* and *line2* here. Again you can ask students, how many angles are formed?. And you can ask them to total them, you can ask them to label them. For labelling, of course, as we saw earlier it is important to label the points- label the leg points of the angle.

You can either show all the angles or you can show them as different sets of linear pairs, as shown here. And that becomes the discussion by itself. And after showing these angles or as in a parallel lesson, you can show them the third line and you can ask them to explore what is the shape that is formed and you can show them the intersection points and you can show the segments. You can also show the side measures, and then we can go on to measure the angles as well. Hopefully, by the time we come to the angle measure here, students would be comfortable with the idea of the counterclockwise measurement of angle and labelling it correctly. So how will you actually label and measure the angle inside the triangle that has been formed by the intersection of three lines? And it is also useful to have this discussion here on what is the minimum number of lines that you would need to form a closed space.

So this becomes the discussion that can go along with these lines and this particular lesson can also be extended to demonstrate the angle sum property of a triangle or the exterior angle-interior angle relationship in a triangle and so on.

Last slide

So far we have seen the concepts as follows:
(i) we have seen how the angle is formed when you rotate a segment
(ii) we have seen the idea of the interior and the reflex angle
(iii) the orientation of the angle itself and
(iv) how the angle value is not decided by the length of the arms and
(v) the idea of the intersection of lines to form a bounded space which is a triangle and introducing the idea of a triangle as a bounded space created by 3 intersecting lines
(vi) and labelling those created objects and exploring the properties of the angles as well.

These are some of the concepts which we have seen through these two sketches. This is by no means exhaustive and you can make any extensions to these concepts based on the context and levels of your class. For further discussion, what we would really like to suggest that the participants do, is to download these underlined files, and play with them, and share in the discussion forum your reflections on what have been your teaching insights in teaching these concepts in your classrooms and how can a tool like this, help in the transaction of these lessons in a better manner.

In our next video, we will be looking at the *concept of similarity*. Before that, I would like to suggest that you download all the Geogebra tutorials that are uploaded in *week 2* and familiarise yourself with the functionalities of Geogebra and create the sketches that have been mentioned in the tutorials.

Closing webcam

This brings us to the conclusion of this video on the pedagogical possibilities of using Geogebra, in mathematics teaching and learning. Please be sure to download the Geogebra files that have been uploaded under *week 2* of this course and share your reflections on how they can support the effective teaching-learning of mathematics, *especially geometry.*
Thank you.