Teaching Mathematics with Technology (TMT)

Video – Linear equation and simultaneous linear equation

Opening webcam/Slide 1

Hello and welcome to week-3 of the "**Teaching Mathematics with the Technology"** course.

Slide 2

In this video, we will explore the topics *line, linear equation, and simultaneous linear equation* in Geogebra. We will use this video to demonstrate that we can use Geogebra as a tool for exploring concepts in mathematics.

We know that if we have a point on a plane, infinite lines can pass through that point. However, if we have two points, one and only one line can pass through it which we will demonstrate in Geogebra.

Geogebra Construction

I am going to construct a point on this plane. And I am now going to construct a line, that passes through the point. So I will select the point for constructing the line, the tooltip says that for a line we need to select two points, which we know. So the minute I select the first point and I move my cursor across the plane I can see that the Geogebra is waiting to construct the line, and it will construct the line the minute I click and select the second point. So the minute I have two points in the plane, I can see that, I can have only one line pass through it. But so long as I have not selected the second point through the first point, I can have infinite lines. It is easy to demonstrate that *through one point infinite lines can pass* and *through two points we can have one and only one line.*

Slide 4

The linear equation is a topic in Algebra, and we learn that it can be expressed in the form :
$ax + by + c = 0$ or $y = mx + c$, *where 'm'* is *slope of the line* and *'c' is the y-intercept.* Geogebra allows us to see the visualization of this linear equation as a line, where the *'x' and 'y' values* satisfying the algebraic equation are *coordinates of points, which are on this line*. The 'x' and 'y' values that satisfy the equation have to be points on the line; and conversely any point on the line, its *x and y coordinates* will satisfy the equation. Let us see this in Geogebra.

Geogebra Construction

Already I have constructed a line, where the *slope is one* and the *y-intercept is zero*. I have used two *sliders- one for slope and one for the y-intercept*. Here the value of the slope is one and the value of *y-intercept is zero,* so I get the equation: y=x. So let me plot a few points on this line, I am going to select the option - *Point-on-Object* and I am going to construct a few points on this line. I have constructed three points and from the *Algebra view* its very clear that all these three points satisfy the equation *y=x;* the *x and y coordinates are equal*.

Now let me define a point on the plane, where the x and y values are equal, and we will see that this point will come on the line itself. So I'll go to the input bar and I'm going to define a point which is having the x and y coordinates as - two and two *(2,2)*. We can see that the new point that I have constructed on this plane- *D=(2,2),* is also on the line. Any point which is not on this line, those coordinates will not satisfy the relationship of *y = x.*

So E and F (which I have constructed) do not satisfy the requirement of y = x, and they are not on the line as well.

Slide 5

We know that the *slope of a line* is defined as *the ratio of the change in y-coordinates over that of x-coordinates of two points on the line.* We can see in Geogebra that when we *increase the value of the slope* the line becomes *steeper to the x-axis*; and as the *value of the slope goes towards zero,* the line becomes *flatter to the x-axis*. Similarly, if the value of the *slope is negative* we can see the *same behaviour*. So we will just see that in Geogebra in the same file.

I am going to change the value of the *slider* from *1 to 2,* and we can see that the new line formed is *steeper to the x-axis.* And as I increase the value of the slider for the slope, the line that is visible is becoming more and more steeper to the x-axis, although, it will never touch the y-axis; the y-axis is a special case where the slope is not defined. Again as the value of the slider for the *slope reduces*, we can see that the line is becoming *flatter to the x-axis*. And finally when we have the value of *slope as zero* and the *constant is also 0*, the equation of the line becomes y=0 which is the x-axis itself.

Now let us look at when the slope is negative, so now we have got a case of negative slope, y=(-x). As the value of the *slope increases or rather as a negative value reduces,* we find that the line becomes *steeper*. So, when we have a case of : *y=2x or y= (- 2x)*, the line's steepness to the x-axis remains the same, except that the line with a positive slope has an upward orientation from left to right, and the line with a negative slope has an upward orientation from right to left. So, *y=2x and y=(-2x) are reflections of each other on the y-axis*.

And likewise we can see that if the *slope is the same* and the *y-intercept changes,* we get a new line which is parallel to the existing line.

So, let me change the value of 'c' alone. When the value of 'c' changes, the y-intercept changes. I get a new line which is parallel to the existing line. So, I can use the feature of Geogebra called *trace* which will allow me to see both the old and the new lines. So, as I change the *y-intercept* now, I can see that I get new lines which are parallel to the old line. This is the geometric reflection of the meaning that the slope remains constant.

Slide 6

Simultaneous linear equations is a topic in Algebra, where we have two equations. When they are consistent we get a value of x and y, and this value of x and y simultaneously solves both the equations.

Slide 7

We can see this '*Simultaneous linear equations'* in Geogebra. It will allow us to see the visualisation of these two linear equations as two lines, where x and y values that satisfy the two equations simultaneously are the co-ordinates of points on both the lines. Now we know that if two lines intersect, they can intersect in only one point. And hence this point whose co-ordinates x and y values satisfy both the equations simultaneously; this point has to be the *point of intersection of the two lines*. We also have a second case where the two equations do not have a solution, that is, no value of x or y can simultaneously satisfy both equations and this will be a case where the lines are parallel. So let us see that in Geogebra.

Geogebra Construction

So I have a line ***y = (-x) + 10***. So, if I have a line which is intersecting this first line we have a line which is **y= x - 8.** When we solve the linear equations *y = x - 8 and y = - x + 10*, you can see that done in the textbox, we will get the value of *x as 9* and the value of *y as one,* which is the x and y co-ordinates of the *point of intersection, E*.

Now when we consider the case of parallel lines, we have the two lines: *y=(-x) + 10* and *y=(-x)+5.* The slope of *both* these equations is minus one (-1). They are parallel to each other and when we try to solve it, we find that they do not have a solution at all.

So, simultaneous linear equations in the case of intersecting lines and the case of parallel lines, we are able to see the visualisation of the property of simultaneous linear equations on the *Geometry view* in Geogebra.

Closing webcam/Last slide

In this way, we can use Geogebra to explore topics in Algebra and Geometry and exchange our conceptual understanding.
Do explore Geogebra.